Recursive Formulas

Objective: To be able to solve finance problems using recursive formulas (difference equations) on the TI-83.

The structure of a recursive formula has two equations:

\[ U_n = aU_{n-1} + b \]

\[ u(nMin) = \]

The \( U_n \) represents the next term and the \( U_{n-1} \) is the previous term in the language of sequences. The group of symbols, \( u(nMin)= \), is the starting value of the sequence.

For our unit on finances, we will adjust this general formula to help us solve problems that would be a little more involved using the formulas from the sections of chapter 10. Remember seeing the quantity of \((1 + i)\) in our formulas? This will be the value for the variable \( a \) and \( b \) is the periodic deposit (or withdrawal) and \( u(nMin) \) represents the starting amount of the balance. So making these changes, we now have the formula that we need.

\[ U_n = (1 + i)U_{n-1} + b \]

\[ u(nMin)=\{\text{starting value}\} \]

Do not be intimidated by the variables used in this formula. Think of \( U_n \) as being the next balance and \( U_{n-1} \) as the previous balance. And the \( u(nMin)= \) will be the amount of money initially in the account.

Now the calculator needs this formula typed in a particular way.

\[ u(n) = (1 + i)u(n-1) + b \]

\[ u(nMin)=\{\text{starting value}\} \]

To use the recursive capability of the TI-83/84, the calculator must be set for sequences. Perform the following steps to setup the calculator to do recursive formulas.

1) Press MODE. Move the cursor to highlight Seq then ENTER as in the display below:

2) Recursive formulas will be entered under \( Y= \). The symbol \( u \) can be found above the 7 key.
Example #1:
This is an example of how to find the future value of a savings account using compound interest. Create a table that will display future values for a $6,000 deposit into a CD at 9% interest compounded monthly.

\[ i = \frac{0.09}{12} = 0.0075 \quad b = 0 \]

The recursive formula becomes \( U_n = (1.0075)U_{n-1} \) with \( u(nMin)=6000 \)

3) Your display under \( Y= \) should look like the following screen shot. Notice that \( nMin = 0 \), so change your calculator to this value. You want this \( nMin \) to always be 0 when working with these finance problems. The symbol \( u \) can be found above the 7 key.

4) To see the table, first press [TBLSET] and setup the TI-83/84 as in the display below.
5) Press 2nd then [TABLE] (above GRAPH). The TI-83/84 shows the following table. Remember to move the cursor onto the number in the second column and read the answer from the bottom of the display. The table can only display up to 6 characters, so read the answer from the bottom of the screen. In this screen display, you would use $6090.34.

<table>
<thead>
<tr>
<th>n</th>
<th>u(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6000</td>
</tr>
<tr>
<td>1</td>
<td>6005</td>
</tr>
<tr>
<td>2</td>
<td>6136</td>
</tr>
<tr>
<td>3</td>
<td>6182</td>
</tr>
<tr>
<td>4</td>
<td>6280.4</td>
</tr>
<tr>
<td>5</td>
<td>6275.1</td>
</tr>
</tbody>
</table>

\[ u(n) = 6090.3375 \]

6) The table will also display any inputted value by going back to [TBLSET] and changing the Indpnt to Ask.

7) Then go to [TABLE] and enter the desired number.

Example #2: This is an example of an annuity problem. Create a table displaying the future values for an account that is opened with $1,000 as an initial deposit. And the interest rate is 6% compounded monthly with a regular monthly deposit of $100 to the account. What would be the value of this account after 15 years?

\[ i = \frac{.06}{12} = .005 \quad b = 100 \]

The recursive formula is \[ U_n = 1.005U_{n-1} + 100 \] with \([nMin]=1000\)

The amount in the account after 15 years (you use \(n = 180\)) is $31535.96
Example #3: This is an example of a loan. A loan of $2,000 at 7.5% interest compounded monthly is to be paid back in one year with 12 monthly payments. Calculate the monthly payment and then on the TI-83 display the amortization schedule. Recall from our text the formula for finding the monthly payment is:

\[
PMT = \frac{P \cdot i}{1 - (1 + i)^{-n}}
\]

where \( i = \frac{.075}{12} = .00625 \) and \( n = 12 \) with \( P = 2000 \)

Calculating, we get \( PMT = 173.51 \)

The value of \( b = -173.51 \) is used since this amount is deducted from the previous balance in the account. So the recursive formula is:

\[
U_n = 1.00625U_{n-1} - 173.51 \quad \text{with } u(N_{\text{Min}}) = 2000
\]

Display the amortization table by using the same procedure as in the previous problems. Be sure to scroll down to \( n=12 \) in your table. Why is there a balance of $.06 remaining? Due to the rounding of the value for the payment, sometimes this last amount does not become $.00. Since there were to be only 12 payments, the last payment would have $.06 added to $173.51. This last payment is usually not the same amount as the other payments. It is adjusted up or down by a small amount so that the final balance becomes exactly $.00. The lending institution will tell what this final amount should be when it becomes time to pay the last payment.

The amortization table is shown below:

Example #4: I borrow $4,000 at 6% interest compounded monthly. I agree to make monthly payment of $300 each until the loan is paid off.

Step 1: Determine the value of \( i \) and write the recursive formula.

Step 2: Enter the recursive formula into the calculator and look at the amortization table.

How many payments are needed?

What is the amount of the last payment?