Objective: To be able to evaluate a function by using the TABLE command on the TI-83/84.

Problem: Auto Emissions. The number of parts per million of nitric oxide emissions \( y \) from a certain car engine is approximated by the model
\[
y = -5.05x^3 + 3857x - 38411.25
\]
where \( x \) is the air-fuel ratio and where \( x \) is \( 13 \leq x \leq 18 \)

Find the nitric oxide emissions for \( x = 14, 15, \) and \( 18 \)

Steps:
1) Press \(^2\) [TBLSET] (above WINDOW). Set the table for values starting with 13 and set all other options as in the display.
   Display:
   
   **TABLE SETUP**
   TblStart=13
   □Tbl=1
   IndPnt: Auto Ask
   Depend: Auto Ask

   2) Enter the equations in \( y_1 \) under Y=
   Display:
   
   **Plot1 Plot2 Plot3**
   \[
   y_1=-5.05x^3+3857x-38411.25
   \]
   \[
   y_2=
   y_3=
   y_4=
   y_5=
   y_6=
   
   3) Press \(^2\) TABLE (above GRAPH).
   Find the solution for \( x = 14 \) (1729.55)
   \( x = 15 \) (2400)
   \( x = 18 \) (1563.15)

   4) A specific value for \( x \) can also be evaluated. Return to **TABLE SETUP** and change the independent variable to **ASK**.
5) Press 2nd TABLE and enter the value for x (for example, x = 16.5) and the value for y is displayed.

Display:

<table>
<thead>
<tr>
<th>x</th>
<th>Y₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.5</td>
<td>2544</td>
</tr>
</tbody>
</table>

6) Enter as many values as needed.
Objective: To be able to use the value command to evaluate a function and view its graph on the TI-83/84.

Problem: The dosage D in milligrams of Ivermectin, a heartworm preventive, for a dog who weighs \( x \) pounds is given by \( D(x) = \frac{136}{25} x \).

Steps:
1) Enter the equations in \( y_1 \) under \( Y= \). The variable \( y \) must replace the variable \( D(x) \). Notice the use of parentheses.

Display:

2) Set the values in the WINDOW. Select appropriate values that will display the equation by

What would be a minimum weight for a dog?
What would be a maximum weight for a dog?
What would be a minimum dosage?
What would be a maximum dosage?
So that we can see each axis we will use:

\[
X_{\text{min}} = -5 \quad Y_{\text{min}} = -5
\]
\[
X_{\text{max}} = 150 \quad Y_{\text{max}} = 1000
\]
\[
X_{\text{sel}} = 10 \quad Y_{\text{sel}} = 100
\]

Note: Other reasonable values are possible for this problem.

3) Press GRAPH.
4) Press \( 2^{\text{nd}} \) CALC (above trace). Select 1: value and press ENTER.
5) Enter a value for \( x \) at the cursor. A possible display is shown below.

Display:
6) To enter another value of \( x \), just type the number and press ENTER. The graphic calculator will display the coordinates and the cursor will locate that point on the graph.

7) The value command will only return a value for \( y \) if the value of \( x \) is between the \( \text{Xmin} \) and \( \text{Xmax} \) values on the WINDOW.
Objective: To be able to enter and graph an equation on the TI-83/84 by using the y= key.

Graph the equation $2x + y = 7$ on the TI-83/84

Steps:
1) Solve the equations for $y$ (that is, change the equation into $y=mx + b$ form of a line). For this example, the equation becomes $y = -2x+7$.

2) Press Y= key. Now enter the equation using $y_1$. Be sure to use the negative sign (-) before the coefficient of $x$. After keying in the equation, press ENTER. Your display should look like this:

3) Press ZOOM. Move the cursor down to highlight 6: ZStandard. Now press ENTER. The TI-83/84 displays a graph of the equation for $x$-values from -10 to 10 and $y$-values from -10 to 10. This command must be used to restore the display after using other options from this menu.

4) The graph can be displayed several ways by using other commands under the ZOOM key. One example is to move the cursor to highlight 4: ZDecimal and then press ENTER. The calculator now displays the following graph for $x$-values from -4.7 to 4.7 and $y$-values from -3.1 to 3.1.

5) Your instructor may demonstrate other commands under ZOOM appropriate for your course.
Objective: To be able to display better graphs on the TI-83/84.

Display the graph of \( y = 25x + 5000 \) on the TI-83/84.

After entering the equation, most users of the TI-83/84 would press ZOOM and select option 6: ZStandard. Unfortunately, due to the size of the numbers in the equation, the graph will not show up on the display. This command will only graph between -10 and 10 inclusive for both the x and y axes which are not large enough to display this equation. Other options, however, are available.

Option 1:
1) First determine the \( x \) and \( y \) intercepts for the equation. In this example, the intercepts are (0, 5000) and (-200, 0).
2) Press WINDOW. Looking at the \( x \) values of the two points, determine the smallest value (\( x = -200 \)). Enter a value for Xmin that is smaller than \( x \), say -210. Next determine the largest value (\( x = 0 \)). Enter a value for Xmax that is a little larger than \( x \), say 10. Now determine a convenient scale based on the two numbers entered. In this case, use 20 and enter this for Xscl. Note: Other values for these three numbers are possible.
A possible “rule of thumb” is to first find the \( x \)-scale by dividing the \( x \)-intercept by 10 and then add/subtract this amount to the Xmax and Xmin.
3) Following the same procedure, determine the Ymin, Ymax, and Yscl values to be entered by using the \( y \) values of the two intercepts. For this example, use the values -100, 5100 and 200. Note: Other values for these three numbers are possible.
Following the “rule of thumb” as outlined before now find the \( y \)-scale by dividing the \( y \)-intercept by 10 and then add/subtract this amount to the Ymax and Ymin. The main idea is to be able to see the graph and all of its intercepts.

<table>
<thead>
<tr>
<th>WINDOW</th>
<th>WINDOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xmin=-210</td>
<td>Xmin=-220</td>
</tr>
<tr>
<td>Xmax=10</td>
<td>Xmax=20</td>
</tr>
<tr>
<td>Xscl=20</td>
<td>Xscl=20</td>
</tr>
<tr>
<td>Ymin=-100</td>
<td>Ymin=-500</td>
</tr>
<tr>
<td>Ymax=5100</td>
<td>Ymax=5500</td>
</tr>
<tr>
<td>Yscl=200</td>
<td>Yscl=500</td>
</tr>
<tr>
<td>Xres=1</td>
<td>Xres=1</td>
</tr>
</tbody>
</table>

4) Now press GRAPH. The graph is displayed below.
5) Notice that the graph contains the axes as well as the two intercept. This option may give the user a better screen to work with when answering questions about the graph.

Option 2:

1) After entering the equation under Y=, press ZOOM and select another command on this menu by moving the cursor down to highlight option 0: **ZoomFit**. Now press ENTER.

2) The calculator will determine a scale that will display the graph of the equation. The display is shown below. However, **ZoomFit** may not show all the intercepts or the axes so you may have to adjust the window for a better display. Just use this option to give you a start on setting a good window.
Objective: To be able to use the intersect command on the TI-83/84 to find the point of intersection.

Find the point of intersection of the system of equations.
\[ 5x - y = 150 \]
\[ 10x + y = 600 \]

Steps:
1) First solve each equation for \( y \). Then enter each equation for Y= using \( y_1 \) and \( y_2 \).

2) Find the \( x \) and \( y \) intercepts for each equation. In this example, they are (0, -150) and (30, 0) for \( y_1 \) and (0, 600) and (60, 0) for \( y_2 \). Using these points, enter these values in WINDOW that are just smaller and just larger for the \( x \) and \( y \) values. For a more detailed explanation, see page A-6 and A-7.

3) Now press 2nd CALC (above the Trace key). Move the cursor down to highlight option \textbf{5: intersect} and press ENTER.

4) The calculator will draw each graph and begin a series of three questions. Just press ENTER after each of the three questions. (This is the fastest way to find the intersection!) The calculator will now display the intersection point and its coordinates.

5) Note: Other options under ZOOM could be used to graph each line. However, this method seems to fit a large variety of different equations and conditions.
Operations with Matrices

Consider the following tables:

TABLE A
Taxable Sales for the Month of June
(Sales in Thousands of Dollars)

<table>
<thead>
<tr>
<th></th>
<th>Waldorf</th>
<th>LaPlata</th>
<th>Lexington Park</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giant</td>
<td>155</td>
<td>89</td>
<td>42</td>
</tr>
<tr>
<td>Safeway</td>
<td>210</td>
<td>88</td>
<td>115</td>
</tr>
<tr>
<td>Food Lion</td>
<td>83</td>
<td>37</td>
<td>31</td>
</tr>
</tbody>
</table>

TABLE B
Taxable Sales for the Month of July
(Sales in Thousands of Dollars)

<table>
<thead>
<tr>
<th></th>
<th>Waldorf</th>
<th>LaPlata</th>
<th>Lexington Park</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giant</td>
<td>129</td>
<td>96</td>
<td>95</td>
</tr>
<tr>
<td>Safeway</td>
<td>185</td>
<td>90</td>
<td>99</td>
</tr>
<tr>
<td>Food Lion</td>
<td>88</td>
<td>58</td>
<td>78</td>
</tr>
</tbody>
</table>

Suppose that you are a State of Maryland employee who has been assigned the task of writing a report for the nine grocery stores in the Southern Maryland area listed in Tables A and B. Among other things your report is to contain tables that show all of the following:

1) Combined sales for June and July.
2) The increase in sales from June to July.
3) Total tax collected at each store for the months of June and July combined.

Enter Table A as matrix \([A]\) and Table B as matrix \([B]\) into the TI-83/84 calculator

1) On the TI-83: Press MATRX key and move the cursor over to EDIT and press ENTER.
   On the TI-84: Press 2nd [MATRX] (above x⁻¹) and move the cursor over to EDIT and press ENTER.
2) Set dimensions and enter values.
3) To exit, press 2nd [QUIT] (above MODE).

For
1) Use the calculator to do \([A] + [B]\)
2) Do \([B] - [A]\)
3) Do \(([A] + [B]) \times .05\)
(Remember sales are in thousands of dollars)
Objective: To be able to solve a system of equations using the TI-83/84.

To solve the following system of equations:

\[
\begin{align*}
5x + 2y &= 20 \\
x - y &= -3
\end{align*}
\]

Steps:

1) Rewrite the system of equations into a matrix equation.
   Recall the matrix equation is \([A] \cdot [X] = [B]\)

\[
\begin{bmatrix}
5 & 2 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
20 \\
-3
\end{bmatrix}
\]

2) Enter the elements of \([A]\).
   Press \([\text{MATRX}]\) key. Use right arrow key, to move to the edit mode by pressing it twice. Press \(\text{ENTER}\) to bring up \([A]\) and change its dimensions if necessary. Press \(\text{ENTER}\) after each number entered. Enter all elements of \([A]\). Now press \(2^{\text{nd}}\) and then \([\text{QUIT}]\) (above \(\text{MODE}\)). This will exit edit mode.

3) Enter the elements of \([B]\).
   Follow the steps as for \([A]\) but \([B]\) instead.

4) Press \(2^{\text{nd}}\) then \([\text{QUIT}]\) (above \(\text{MODE}\)). This will exit edit mode.

Remember the solution of this system can be found by using:

\([X] = [A]^{-1} \cdot [B]\)

5) Using a clear display, we need to find \([A]^{-1}\). Press \([\text{MATRX}]\) key and then press \(\text{ENTER}\). The symbol for the matrix is displayed. Next press \(x^{-1}\) key to now show the symbol for the inverse of the matrix.

6) Press the multiplication key.

7) Now display the second matrix by pressing \([\text{MATRX}]\), down to option 2 and then press \(\text{ENTER}\).

8) Press \(\text{ENTER}\) and the solution is displayed. The point of intersection is (2, 5).

Note: If an error occurs recheck the values entered for matrices \([A]\) and \([B]\). If error still exists, remember not all systems of equations have a unique solution. Find these solutions by solving each equation for \(y\) and check he graphs of that system.
1) The properties of pure iron can be altered by adding other elements to form iron alloys. Elements that are often added are carbon, chromium, silicon, and nickel. Three iron alloys contain the following percents of carbon, chromium, and iron.

<table>
<thead>
<tr>
<th></th>
<th>Alloy X</th>
<th>Alloy Y</th>
<th>Alloy Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>1%</td>
<td>1%</td>
<td>4%</td>
</tr>
<tr>
<td>Chromium</td>
<td>0%</td>
<td>15%</td>
<td>3%</td>
</tr>
<tr>
<td>Iron</td>
<td>99%</td>
<td>84%</td>
<td>93%</td>
</tr>
</tbody>
</table>

You have 15 tons of carbon, 39 tons of chromium, and 546 tons of iron. How much of each type of alloy can you make?

Model the situation by writing 3 linear equations:

- Carbon
- Chromium
- Iron

Use the TI-83/84, and find the solution to your system.

2) Gold jewelry is seldom made of pure gold because pure gold is soft and expensive. Instead, gold is mixed with other metals to produce a harder, less expensive gold alloy. The amount of gold (by weight) in an alloy is measured in karats. Anything made of 24-karat gold is 100% gold. An 18-karat gold mixture is 75% gold and so on. Three gold alloys contain the percents of gold, copper, and silver shown in the table. You have 20,144 grams of gold, 766 grams of copper, and 1,990 grams of silver. How much of each alloy can you make?

<table>
<thead>
<tr>
<th>Percent by Weight</th>
<th>Alloy X</th>
<th>Alloy Y</th>
<th>Alloy Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>94%</td>
<td>92%</td>
<td>80%</td>
</tr>
<tr>
<td>Copper</td>
<td>4%</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Silver</td>
<td>2%</td>
<td>6%</td>
<td>16%</td>
</tr>
</tbody>
</table>
3) A company produces three combinations of mixed vegetables that sell in 1-kg packages. Italian style combines 0.3 kg of zucchini, 0.3 kg of broccoli, and 0.4 kg of carrots. French style combines 0.6 kg of broccoli and 0.4 kg of carrots. Oriental style combines 0.2 kg of zucchini, 0.5 kg of broccoli, and 0.3 kg of carrots. The company has a stock of 16,200 kg of zucchini, 41,400 kg of broccoli, and 29,400 kg of carrots. How many packages of each style should it prepare to use all existing supplies?

4) A lake is stocked each spring with three species of fish, A, B, and C. Three foods I, II, and III are available in the lake. Each fish of species A requires an average of 1.32 units of food I, 2.9 units of food II, and 1.75 units of food III each day. Species B fish requires an average of 2.1 units of food I, 0.95 units of food II, and 0.6 units of food III each day. Species C fish requires an average of 0.86 units of food I, 1.52 units of food II, and 2.01 units of food III each day. If 490 units of food I, 897 units of food II, and 653 units of food III are used each day, how many fish of each species should be stocked?
Supplemental Problems for System of Equations

1) Sony produces three types of televisions. The Plasma TVs requires 2 hours of soldering, 2 hours of assembly, and 1 hour of testing time. The LCD TVs requires 1 hour of soldering, 3 hours of assembly, and 1 hour of testing time. The Projection TVs requires 3 hours of soldering, 2 hours of assembly, and 2 hours of testing time. In the factory, there are 100 hours of soldering time, 100 hours of assembly time, and 65 hours of testing time available per shift. How many of each model should be produced to effectively utilize all of the allocated time?

2) A student purchased textbooks, paperbacks, and notebooks at the bookstore. Textbooks weigh 2 lb. each and cost $120.00. Paperbacks weigh 1 lb and cost $17.50 each. Notebooks weigh $\frac{1}{2}$ lb and cost $7.50 each. The number of notebooks is the same as the number of textbooks and paperbacks combined. The purchase weighed 19 lbs and cost $660.00. How many textbooks, notebooks, and paperbacks did she buy?

3) Ed Smith, who was building a shed for his lawn mower, went to the lumber yard and bought one pound each of the three kinds of nails: small, medium and large. After doing part of the work, Ed found that he had underestimated the number of small and large nails he needed. So he went back to the lumber yard and bought another pound of the small and 2 pounds of the large nails. Later in the week, he again started running out of small and medium nails. He went to the same lumber yard and bought another pound each of the small and medium nails. Upon looking over his bills he noted that on the first trip for nails he spent $10.50. The second trip he spent $11.50 and the last trip was $6.00. What is the price per pound for each size of nail?
Linear Programming Problems
A graphing calculator can assist with linear programming problems by graphing the outline of a feasible set and using the INTERSECT function to find the corner points. The shading of the feasible set can be done on the TI-83/84, however it is probably more time effective to graph the lines by using the intercepts and determine which points are the corner points by inspection.

Recall the procedure for graphing lines from a lesson we did earlier in the semester. Lines were entered from \( y = mx + b \) form using the Y= function. Scales were set using intercepts and the WINDOW function. And the coordinates of the points of intersection were found by using the INTERSECT function.

Finally note that for the most part your graphs will be confined to the first quadrant. Adjust your scales so that \( X_{\text{min}} = -5 \) and \( Y_{\text{min}} = -5 \). (This will retain the picture of the axis but take up less space from the first quadrant). Adjust the rest of the scales according to the problem by using the intercepts. Refer to A-6 and A-7 from previous work).

Suppose the following is the system of constraints for some linear programming problem.

\[
\begin{align*}
3x - 7y & \geq -24 \\
2x + 3y & \leq 14 \\
x & \geq 0, \quad y \geq 0
\end{align*}
\]

Also, \( P = 8x + 5y \) is the objective function that is to be maximized.

Here is a step-by-step outline that can be followed to solve linear programming problems with assistance from the TI-83/84 to do the tedious portion of locating the lines and points of intersection.

1) Put all constraints in \( y = mx + b \) form. And find the x and y intercepts for all inequalities
2) Decide on the values for the WINDOW by using the x and y intercepts.
3) Using \( Y_1 \) and \( Y_2 \) etc. graph all the lines.
4) Mentally analyze the position of the shading (or use a test point).
5) Choose the points from the graph that will form the corner points of the feasible set.
6) Using the INTERSECT command on the calculator determine the coordinates of the corner points.
7) Substitute those points into the objective function to answer the problem.
1) A nutritionist advises that an individual who is anemic take at least 2400 mg of iron, 2100 mg of Thiamine and 1500 mg of Riboflavin over a period of time. Two brands of medication are available that contain the needed supplements. Brand A costs $0.06 per pill and has 40 mg of iron, 10 mg of Thiamine and 5 mg of Riboflavin. Brand B costs $0.08 per pill and has 10, 15 and 15 mg of the same supplements respectively. How many of each pill must this person purchase in order to meet the requirements of the diet but keep the purchase to the lowest cost?

Use this outline for assistance:
   Establish the variables precisely:

   Create a table:

<table>
<thead>
<tr>
<th>Pills</th>
<th>Iron</th>
<th>Thiamine</th>
<th>Riboflavin</th>
</tr>
</thead>
</table>

   Write all the constraints:

   Write the objective function:

   Rewrite your constraints from above in a calculator ready format. For the time being, we will treat these inequalities as equations.

   Now find the x and y intercepts for \( y_1, y_2 \) and \( y_3 \).

   Using the calculator, draw the graph of the feasible set, and then determine which side of each line will satisfy the inequality and finally determine the coordinates of the corner points. Hint: There are four for this problem.

   Substitute these into your objective function. The solution is _____________

2) A 4-H member raises geese and pigs as a 4-H project. She has decided to raise no more than 16 animals, including no more than 10 geese as they can be cantankerous. She spends $5 to raise a goose and $15 to raise a pig per week and has $180 budgeted per week for this project. Upon completing the project she hopes to sell the animals for a profit. Each goose will bring a profit of $60 at it sale and each pig will bring a profit of $200. She wants to calculate how many of each of these animals she should raise to earn the greatest profit.
Finance Formulas

List of Variables

- \( r \) is the annual interest rate, as a decimal
- \( i \) is the interest rate per period
- \( t \) is the number of years
- \( m \) is the number of periods (payments) per year
- \( n \) is the total number of periods (payments)

\( P \) is the
- Principal using simple interest,
- Present value using compound interest
- Present value of an annuity or remaining amount of a loan

Note: Our text uses an upper case \( P \) to represent the principal.

\( A \) is the
- Future value of a fund using simple interest
- Future value of a fund using compound interest

\( \text{PMT} \) is the
- Payment to an annuity
- Payment of a loan

Note: Our text uses a lower case \( p \) to represent the payment.
Finance Formulas

Simple Interest

Interest     \[ I = Prt \] or     \[ I = A - P \]

Future Value     \[ A = P + Prt \] or     \[ A = P(1 + rt) \]

\[ A = P + I \]

Principal     \[ P = \frac{A}{(1+rt)} \]

Compound Interest

Interest     \[ i = \frac{r}{m} \]

\[ \text{APR} = (1+i)^m-1 \]

Number of periods \[ n = m \times t \]

Compound Interest

Future Value     \[ A = P(1 + i)^n \]

Present Value     \[ P = \frac{A}{(1+i)^n} \]

Annuities

Future Value     \[ A = \text{PMT} \times \frac{[(1+i)^n - 1]}{i} \]

Present Value     \[ P = \text{PMT} \times \frac{[1-(1+i)^{-n}]}{i} \]

Payment to a     \[ \text{PMT} = A \times \frac{i}{[(1+i)^n - 1]} \]

Sinking Fund

Amortization of Loans

Future Value     \[ A = \text{PMT} \times \frac{[(1+i)^n - 1]}{i} \]

Present Value     \[ P = \text{PMT} \times \frac{[1-(1+i)^{-n}]}{i} \]

Payment     \[ \text{PMT} = P \times \frac{i}{[1-(1+i)^{-n}]} \]
Recursive Formulas

Objective: To be able to solve finance problems using recursive formulas (difference equations) on the TI-83.

The structure of a recursive formula has two equations:

\[ U_n = aU_{n-1} + b \]

\[ u(nMin) = \]

The \( U_n \) represents the next term and the \( U_{n-1} \) is the previous term in the language of sequences. The group of symbols, \( u(nMin) = \), is the starting value of the sequence.

For our unit on finances, we will adjust this general formula to help us solve problems that would be a little more involved using the formulas from the sections of chapter 10. Remember seeing the quantity of \((1 + i)\) in our formulas? This will be the value for the variable \( a \) and \( b \) is the periodic deposit (or withdrawal) and \( u(nMin) \) represents the starting amount of the balance. So making these changes, we now have the formula that we need.

\[ U_n = (1 + i)U_{n-1} + b \]

\[ u(nMin) = \{\text{starting value}\} \]

Do not be intimidated by the variables used in this formula. Think of \( U_n \) as being the next balance and \( U_{n-1} \) as the previous balance. And the \( u(nMin) = \) will be the amount of money initially in the account.

Now the calculator needs this formula typed in a particular way.

\[ u(n) = (1 + i)u(n - 1) + b \]

\[ u(nMin) = \{\text{starting value}\} \]

To use the recursive capability of the TI-83/84, the calculator must be set for sequences. Perform the following steps to setup the calculator to do recursive formulas.

1) Press MODE. Move the cursor to highlight Seq then ENTER as in the display below:

2) Recursive formulas will be entered under Y=. The symbol u can be found above the 7 key.

A-18
Example #1:
This is an example of how to find the future value of a savings account using compound interest. Create a table that will display future values for a $6,000 deposit into a CD at 9% interest compounded monthly.

\[ i = \frac{0.09}{12} = 0.0075 \quad b = 0 \]

The recursive formula becomes \( U_n = (1.0075)U_{n-1} \) with \( u(nMin)=6000 \)

3) Your display under \( Y= \) should look like the following screen shot. Notice that \( nMin = 0 \), so change your calculator to this value. You want this \( nMin \) to always be 0 when working with these finance problems. The symbol \( u \) can be found above the 7 key.

4) To see the table, first press [TBLSET], and setup the TI-83/84 as in the display below.
5) Press 2nd then [TABLE] (above GRAPH). The TI-83/84 shows the following table. Remember to move the cursor onto the number in the second column and read the answer from the bottom of the display. The table can only display up to 6 characters, so read the answer from the bottom of the screen. In this screen display, you would use $6090.34.

<table>
<thead>
<tr>
<th>n</th>
<th>u(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6000</td>
</tr>
<tr>
<td>1</td>
<td>6045</td>
</tr>
<tr>
<td>2</td>
<td>6136</td>
</tr>
<tr>
<td>3</td>
<td>6228.4</td>
</tr>
<tr>
<td>4</td>
<td>6329.1</td>
</tr>
</tbody>
</table>

\[ u(n) = 6090.3375 \]

6) The table will also display any inputted value by going back to [TBLSET] and changing the Indpnt to Ask.

7) Then go to [TABLE] and enter the desired number.

Example #2: This is an example of an annuity problem. Create a table displaying the future values for an account that is opened with $1,000 as an initial deposit. And the interest rate is 6% compounded monthly with a regular monthly deposit of $100 to the account. What would be the value of this account after 15 years?

\[ i = \frac{.06}{12} = .005 \quad b = 100 \]

The recursive formula is \[ U_n = 1.005U_{n-1} + 100 \] with \( u(nMin) = 1000 \)

The amount in the account after 15 years (you use \( n = 180 \)) is $31535.96
Example #3: This is an example of a loan. A loan of $2,000 at 7.5% interest compounded monthly is to be paid back in one year with 12 monthly payments. Calculate the monthly payment and then on the TI-83 display the amortization schedule. Recall from our text the formula for finding the monthly payment is:

\[ \text{PMT} = \frac{P \cdot i}{[1 - (1 + i)^{-n}]} \]

where \( i = \frac{.075}{12} = .00625 \) and \( n = 12 \) with \( P = 2000 \)

Calculating, we get \( \text{PMT} = \$173.51 \)

The value of \( b = -173.51 \) is used since this amount is deducted from the previous balance in the account. So the recursive formula is:

\[ U_n = 1.00625U_{n-1} - 173.51 \]  

with \( U(\text{Min}) = 2000 \)

Display the amortization table by using the same procedure as in the previous problems. Be sure to scroll down to \( n=12 \) in your table. Why is there a balance of $.06 remaining? Due to the rounding of the value for the payment, sometimes this last amount does not become $.00. Since there were to be only 12 payments, the last payment would have $.06 added to $173.51. This last payment is usually not the same amount as the other payments. It is adjusted up or down by a small amount so that the final balance becomes exactly $.00. The lending institution will tell what this final amount should be when it becomes time to pay the last payment.

The amortization table is shown below:

Example #4: I borrow $4,000 at 6% interest compounded monthly. I agree to make monthly payment of $300 each until the loan is paid off.

Step 1: Determine the value of \( i \) and write the recursive formula.

Step 2: Enter the recursive formula into the calculator and look at the amortization table.

How many payments are needed?

What is the amount of the last payment?
Recursive Formulas II

1) Tom, for his retirement, started an annuity with payments of $150 every quarter. The interest rate is 6.8% compounded quarterly.
   a) What would the balance be after 20 years?
   b) After seeing this number, Tom felt that this amount of money was not going to be enough for his retirement, so he decides to initially deposit $3,000 into the annuity and then contributes $150 every quarter for the next 20 years. How much is in the account after the 20 years? How much interest was earned during the 12th year of the annuity?
   c) After 20 years, Tom no longer makes deposits but his account is still earning the same amount of interest. How much is in the account after 5 more years?

2) Kathy and David wanted to start saving for their son's, Eli, college fund. They decide they would like to have saved $15,000 in 16 years. The interest rate is 7.6% compounded quarterly.
   a) What is their payment? How much is in Eli's college fund after 10 years?
   b) Eli's godfather decided he would help too. But he wanted to make a lump sum deposit that would grow to $15,000 at 7.6% interest compounded quarterly. What is his payment? How much is in the fund after 10 years?

3) Janet and Jeff bought a house for $220,000. Their down-payment was 10% of the purchase price. Their mortgage is for 30 years at 6.18% interest compounded monthly.
   a) Find their monthly payment.
   b) How much interest is paid during the 10th year of the mortgage?
   c) After 15 years of making payments, Janet and Jeff decide to add an additional $100 to their monthly payment. They did this for the next 5 years, so what is their balance at the end of the five years?
   d) Again Janet and Jeff decide to add an additional $50 to their previous monthly payment. So after 2 more years of doing this what is the balance now? How much equity do they now have in their house?
   e) Janet and Jeff continue to make the same payment. What is the amount of the last payment? How many payments, from the beginning, were necessary until the balance is $0?
Mathematics of Personal Finance
The following two problems are to be used for discussion purposes to allow your instructor to develop concepts not specifically addressed in our text. Specifically we will look at the finances of buying a car in Maryland and some of the "pitfalls" of a home mortgage. Use these questions as additional class notes for the Finance Unit:

I have found my "dream" car. It is a new luxury model loaded with options that has a window price of $34,400. The salesman said he was in a good mood today and would deduct $2400 from this price if I bought the car today. I am trading in my old car that has a book value of $6500. The State sales tax is 5% and the license tag fee is $70. The manufacturer is offering a $500 rebate, which I will apply to the purchase price of the car. The State of Maryland also collects $2 for a fund to help dispose and cleanup old tire dumps. After making a cash down payment of $3,000, I will borrow the remainder of the amount necessary to purchase this car.

a) How much do I need to borrow to buy this car?

b) If the bank is charging 7.2% interest compounded monthly for the loan on a five-year loan, what is my monthly payment?

c) After owning the car for 3 years you decide to buy another car. The salesman at the dealership tells you that you can trade in your present car and the dealership would pay off the rest of your loan. However, the **negative equity** would carry over to the amount that is needed to finance the new car. How much is this **negative equity**?
Ted and Sue are going to purchase a condo in Ocean City. They have selected a two-bedroom model one block from the beach that costs $590,000. Also because the market is flat on sales, the builder has agreed to pay all closing costs and pre-paid items at settlement. The mortgage company requires a 20% down payment which Ted and Sue have available. And they will finance the remainder of the purchase price by taking out a mortgage at 6.12% interest.

a) What is their monthly payment if they choose a 20-year loan?

b) How much total interest will they end up paying over the 20 years?

c) After exactly 12 years of owning and using this condo, Ted and Sue have decided to sell it to have cash available for their children's college education. What is the payoff on this loan after exactly 12 years of making payments?

d) Inflation in the Ocean City housing market has averaged 5% per year over the 12 years Ted and Sue have owned this condo. How much equity do they have in this condo that can now be used for college expenses?
Mathematics of Personal Finance II  
**Credit Cards and Using a Recursive Formula**

According to a report on consumer buying, the average credit card user carries an average balance of $2500 from one month to the next. Of course, the consumer must pay interest on this amount. The next set of questions will show how easy it is for people to get in “over their head” financially.

The new balance is affected by the previous balance, new purchases, the number of days in the billing cycle and the interest rate offered by the credit card company. Since the number of days in the billing cycle will vary from one month to the next, the credit card company computes an average daily balance and a daily periodic rate. The daily periodic rate is equal to the annual percentage rate divided by 365.

Suppose MasterCard's annual percentage rate (APR) is 19.8%. Compute the daily periodic rate to five decimal places.

\[ \text{\%} \]

Since the monthly balance has many factors contributing to its amount, the circumstances will be simplified so that some final conclusions can easily be made. This will allow the use of a recursive formula to approximate the use and pitfalls of a credit card.

1) Suppose you owe $614.57 on purchases for the month. And the credit card company computes your minimum payment to be $13. You decide to just pay the $13 every month until the balance is paid. If you make no new purchases, and the APR is 19.8% complete the following:

\[ i = \frac{\text{\%}}{12} = \text{\%} \]

Write the recursive formula:

\[ U_n = \text{\%} \quad u(n\text{Min}) = \text{\%} \]

For the 1st payment, how much of the $13 was interest and how much went to reducing the balance?

Enter the recursive formula into the calculator and find the amount remaining on the balance after making this first payment.

What is the balance after 1 year? \[ \text{\%} \]
after 2 years? \[ \text{\%} \]
after 5 years? \[ \text{\%} \]

For the 60th payment, how much of the $13 was interest and how much went to reducing the balance?
What is the amount of the last payment?

What was the total amount repaid? And how much was interest?

2) Earlier in this supplement it was noted that the average consumer carries a balance of $2500 every month. Again assume no new purchases and you are just going to pay the minimum payment of $52.88 each month. The annual percentage rate is 18.9%

\[ i = \text{___________} \]

Write the recursive formula:

\[
\text{______________________________________________________________}
\]

What is the balance after 1 year? ________________
3 years? ________________
6 years? ________________

For the 50th payment, how much is interest and how much went to reducing the balance?

What is the amount of the last payment?

What was the total amount repaid and how much of it was interest?
Logic Introduction

In this unit, we will learn some basic concepts about logic and how it is applied in mathematics and everyday situations.

A statement (some books may use the word “propositions”) is a declarative sentence whose truth-value can be determined. When the phrase truth-value is used, we mean that a statement is always true or that it is always false. In logic, we deal with absolute concepts where there are no shades of gray or chance of confusion about some situation. In computer science, when something is true it has a numerical value of 1 and if it is false it has a numerical value of 0. In everyday life, a light bulb is either on or off. So if it is on, or completes an electrical circuit, it has a truth-value of 1.

In our study of logic, the words “statement” and “sentence” are not really interchangeable as they may be in writing English compositions. A sentence becomes a statement only if it has a particular truth-value (that is, we can determine that it is always true or always false). For example, suppose that there are witnesses to a crime. These witnesses will go to the police and make statements to what they saw. These sentences become sworn “statements” in the investigation of this crime.

Let’s look at a few examples. Determine which of the following problems are statements.

1) Phil has a freckle on his face.

2) Hickory, Dickory, Dock, the mouse ran up the clock.

3) \(3 + 7 = 11\)

4) The date, October 11, 1950, was on a Tuesday.

5) Is he handsome?

6) Thomas Jefferson was the third President of the United States.

7) Close the door!

8) \(13 - 5 \neq 6 + 2\).

9) \(x + 6 > 12\).

10) Do you have a cold?

The following statement is a good summary of what this unit on logic is about.

*The building blocks of logic are statements, connectives, and rules for calculating the truth of compound sentences.*
CONNECTIVES

Connectives (or conjunctions that you know from an English grammar class) are used to combine two or more simple statements into a larger compound statement. For example, we would write the compound sentence:

Tom and Joe went to the store.

That would be a better-constructed statement than the two simple sentences of:

Tom went to the store.
Joe went to the store.

The word “and” is used to connect these two sentences together.

The mathematical symbols and their meanings are listed below.

\( \land \) is "and"
\( \lor \) is "inclusive or"
\( \oplus \) is "exclusive or"
\( \rightarrow \) is "implies"
\( \iff \) is "equivalent to"

Another symbol that is used but is not a connective is the word “not” or “negation”. This is used when making a statement have the opposite truth-value. The symbol looks like ~. Some people call this a “squiggle”.

When working with logic statements, the variables of P, Q and R are traditionally used to represent these statements just in the same way that we commonly use x, y and z to represent numbers when solving equations.

Let P: John is a business major. Q: John is a sophomore in college. R: John has a part-time job

Translate the statements into symbols.
1) John is a sophomore majoring in business.
2) John is not a sophomore in college and is not a business major.
3) If John is a sophomore in college then he has a part-time job.
4) John has a part-time job or he is a business major.
5) It is not true that John has a part-time job or he is a business major.

Let P: Every employee must complete a W-4 form. Q: Every employee is required to be fingerprinted.

Translate the following statements into an English statement.
1) \( P \lor Q \)  
2) \( P \land Q \)  
3) \( \sim P \land \sim Q \)  
4) \( Q \rightarrow P \)  
5) \( \sim [P \lor Q] \)  
6) \( \sim P \rightarrow Q \)
# Truth Tables

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \land Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \lor Q</th>
</tr>
</thead>
<tbody>
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<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
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<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \rightarrow Q</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>T</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \oplus Q</th>
</tr>
</thead>
<tbody>
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<td>T</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \leftrightarrow Q</th>
</tr>
</thead>
<tbody>
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<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Tables II

The textbook covers the truth tables for P “and” Q, P “or” Q, and “not” P. In this section, we will learn some basic concepts about implications, equivalence and the “exclusive or” and how it is applied in mathematics and everyday situations. The main mechanism for determining the truth value (whether a statement is true or false) of our previous statements is found by applying the truth tables. These truth tables were developed by Aristotle and other Greeks to help them in their discussions about law and topics of the day. These rules of conversation were important then as they are today in helping people hold clear and meaningful conversations. A great deal of the logic used in these tables help lawyers argue cases and develop clear and concise contracts. Computer science also uses them in writing lines of programming to control the flow of information.

We will begin by looking at the conjunction, implication, which involves statements that are usually in a "If-then" format. Carefully look at the table below:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P → Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

This is the only conjunction in which the order of the statements is critical to the correct truth value for the compound statement. Notice the second and third lines of this table in which switching the order will give you a different answer. The next connective is called equivalence and its table is displayed below:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P ↔ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

A good example of this truth table is the "Match Game". If the statements P and Q have the same truth value (both true or both false) then the truth value of the compound statement will be true. If they are different, then the truth value is false. The concept of positive and negative numbers when using multiplication is another application of this truth table. Look at the different cases below:

Positive x Positive = Positive
Positive x Negative = Negative
Negative x Positive = Negative
Negative x Negative = Positive
If you replace the words "positive and negative" with the signs of "+ and -", you will see that this is the laws used when multiplying signed numbers.

The last truth table is the "exclusive or" connective. Compare this table to the $P \lor Q$ truth table involving "inclusive or".

| P | Q | $P \oplus Q$
|---|---|---
| T | T | F
| T | F | T
| F | T | T
| F | F | F

Think of "exclusive or" as "one or the other but not both" thinking. When this is compared to the second truth table, you may have noticed that it is identical except for the first line. In the "exclusive or", the first line is false whereas the first line in the other table it is true. Suppose that:

P: I will go to LaPlata        Q: I will stay home

So the compound statement could be written as "I will go to LaPlata or I will stay home". In this situation, you can do either one but not both at the same time. This is the logic used when using "exclusive or".

An application of this type of thinking would be when you go to a fine restaurant and order a meal. If you order a meal you may have a choice of soup or salad with this meal. This is the thinking behind "exclusive or" since you can have soup or salad but not both (unless you pay extra for this privilege).
Supplement to Sets

Let \( U = \{\text{all people}\} \), \( S = \{\text{people who like strawberry ice cream}\} \), \( V = \{\text{people who like vanilla ice cream}\} \), \( C = \{\text{people who like chocolate ice cream}\} \). Describe the following sets using set notation.

1) \( \{\text{people who don’t like strawberry ice cream}\} \)

2) \( \{\text{people who like vanilla but not chocolate ice cream}\} \)

3) \( \{\text{people who like vanilla or chocolate but not strawberry ice cream}\} \)

4) \( \{\text{people who don’t like any of the three flavors of ice cream}\} \)

5) \( \{\text{people who don’t like neither chocolate nor vanilla ice cream}\} \)

6) \( \{\text{people who like only strawberry ice cream}\} \)
Venn diagram of Blood Types

1) Human blood can be classified into types of A, B, AB or O. Furthermore, each of these types is classified as positive or negative depending on whether the antigen called Rh is present. Also note that O blood type refers to the absence of either the A or B blood types.

<table>
<thead>
<tr>
<th>Antigen Combination</th>
<th>Number of Patients</th>
</tr>
</thead>
<tbody>
<tr>
<td>A antigen</td>
<td>60</td>
</tr>
<tr>
<td>B antigen</td>
<td>50</td>
</tr>
<tr>
<td>Rh antigen</td>
<td>65</td>
</tr>
<tr>
<td>A and B antigen</td>
<td>15</td>
</tr>
<tr>
<td>B and Rh antigen</td>
<td>20</td>
</tr>
<tr>
<td>A and Rh antigen</td>
<td>30</td>
</tr>
<tr>
<td>All three antigens</td>
<td>5</td>
</tr>
<tr>
<td>None of the antigens</td>
<td>10</td>
</tr>
</tbody>
</table>

a) How many patients were surveyed?

b) How many patients have AB+ blood?

c) How many patients have A+ blood?

d) How many patients have O- blood?

e) How many patients have exactly one antigen?
Blood Antigens

Objective: To be able to determine the probabilities of different blood types.

Human blood can contain either the A antigen, the B antigen, both the A and B antigens, or no antigens. A third antigen, called the Rh antigen, is important in human reproduction, and again may or may not be present in an individual. Blood is called type A-positive if the individual has the A and Rh, but not the B antigen. A person having only A and B antigens is said to have type AB-negative blood. A person having only the Rh antigen has type O-positive blood. A Venn diagram can be used to display all the types of blood.

The following list shows the percentage of people in the United States with a particular blood type. These percentages may vary in certain sections of the country depending upon the cultural make up of the population.

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>O Positive</td>
<td>38.4%</td>
</tr>
<tr>
<td>A Positive</td>
<td>32.3%</td>
</tr>
<tr>
<td>B Positive</td>
<td>9.4%</td>
</tr>
<tr>
<td>O Negative</td>
<td>7.7%</td>
</tr>
<tr>
<td>A Negative</td>
<td>6.5%</td>
</tr>
<tr>
<td>AB Positive</td>
<td>3.2%</td>
</tr>
<tr>
<td>B Negative</td>
<td>1.7%</td>
</tr>
<tr>
<td>AB Negative</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

Convert the above percentages into decimals and place them in the proper regions of the Venn diagram. Then complete the following questions.

1) Which is the rarest type of blood? ________________

2) What is the probability that a person has type O blood? __________________

3) What is the probability that a person has a positive Rh antigen? ________________

4) What is the probability that a person has type B blood?  ___________________

5) Add the probabilities for all the blood types. What is the total? ___________

This value should be exactly 1, what could account for this discrepancy?

Source: The American National Red Cross

A-34
**Combinations and Permutations**

Decide whether you would use the fundamental counting principle, a permutation or a combination to solve the following problems.

1) A bag of candy contains 12 pieces of candy. In how many ways can 5 pieces be selected?

2) If the Senate is to form a new committee of 5 members, in how many different ways can the committee be chosen if all 100 senators are available to serve on this committee?

3) The license plates in a certain state consist of 3 letters followed by 3 nonzero digits. How many such licenses are possible?

4) In how many ways can two kings be drawn from a deck of cards?

5) There are three boys and three girls at a party. In how many ways can they be seated in a row if they must sit alternating boy, girl, boy, girl?

6) In how many ways can a heart flush be obtained? (A heart flush is a hand of five hearts).

7) At Artist's Dance Studio, every man must dance the last dance. If there are five men and eight women, in how many ways can dance couples be formed for the last dance?

8) A shipment of a hundred TV sets is received. Six sets are to be chosen at random and tested for defects. In how many ways can the six sets be chosen?

9) If there are ten people in a club, in how many ways can they choose a dishwasher and a bouncer?

10) In how many ways can you be dealt two cards from an ordinary deck of cards?

11) In how many ways can five taxi drivers be assigned to six cars?

12) A night watchman visits 15 offices every night. To prevent others from knowing when he will be at a particular office he varies the order of his visits. In how many ways can this be done?

13) How many numbers between 1000 and 9999, inclusive,
    (a) contain no zeros?    (b) contain no ones?    (c) begin with an even digit and end with an odd digit?

14) A lock has a dial with 50 numbers on it. To open it, you must turn left to a number, right to a number, then left to a number. How many possibilities are there if
    (a) the 3 numbers must be different
    (b) the 3 numbers are not necessarily different?

A-35
15) A railway has 30 stations. On each ticket, the departure station and the destination station are printed.
   a) How many different kinds of tickets are there?
   
   (b) If a ticket could be used in either direction between two stations, how many different kinds of tickets would be needed?

16) If you have a $1 bill, a $5 bill, a $10 bill, and a $20 bill, how many different sums of money can you make using one or more of these bills?

17) The Pizza Place offers pepperoni, mushrooms, sausages, onions, anchovies, and peppers as toppings for their regular plain pizza. How many different pizzas can be made?

18) How many 5-digit numbers contain at least one 3? (Hint: How many contain no 3's?)

19) A teacher must pick 3 high school students from a class of 30 to prepare and serve food at the junior high school picnic. How many selections are possible?

20) How many 6-letter "words" can be formed by using all of the letters of
   (a) the word RADISH?
   (b) the word SQUASH?

21) A club with 42 members wants to elect a president, vice president, and a treasurer. From the other members, an advisory committee of 5 people is to be selected. In how many ways can this all be done?

22) A town council consists of 8 members including the mayor.
   (a) How many different committees of 4 can be chosen from this council?
   
   (b) How many of these committees will include the mayor?
   
   (c) How many will not include the mayor?
   
   (d) Verify that the answer to part (a) is the sum of the answers to part (b) and part (c).

23) There are 3 roads from town A to town B, 5 roads from town B to town C, and 4 roads from town C to town D. How many ways are there to go from A to D via B and C? How many different round trips are possible?

24) (a) How many 4-letter "words" can be formed by using the 8 letters of TRIANGLE?
   (b) How many of the "words" formed in part (a) have no vowels?
   (c) How many of the "words" formed in part (a) have at least one vowel?
A popular game of chance in Maryland is KENO. KENO is played with 80 numbers from which 20 are selected for each game. The player can choose how many numbers (spots) from 1 to 10 are played on each game. The player marks the slip, pays the KENO agent and then watches the KENO monitor for the winning numbers for that game. If the player matches some or all of the winning numbers, the player wins money based on the payout chart. On the back of the play slip is the KENO Payout Chart. Determine the probability for some of the payouts from the chart. Some of the problems are started to help you in understanding the pattern. When necessary, probabilities should be rounded to the nearest ten-thousandths.

a) **1 Spot Game**  
   **Match 1 wins**  
   $2

\[
\frac{\binom{20}{1} \times \binom{60}{0}}{\binom{80}{10}}
\]

Probability of winning some money is _________________

b) **2 Spot Game**  
   **Match 2 wins**  
   $10

\[
\frac{\binom{20}{2} \times \binom{60}{8}}{\binom{80}{10}}
\]

Probability of winning some money is _________________

c) **3 Spot Game**  
   **Match 3 wins**  
   $25

\[
\frac{\binom{20}{3} \times \binom{60}{7}}{\binom{80}{10}}
\]

\[
\frac{\binom{20}{2} \times \binom{60}{8}}{\binom{80}{10}}
\]

Probability of winning some money is _________________
d) 4 Spot Game
Match 4 wins $50
Match 3 wins $5
Match 2 wins $1

\[
\frac{\binom{20}{\_} \times \binom{60}{\_}}{\binom{80}{\_}}
\]

\[
\frac{\binom{20}{\_} \times \binom{60}{\_}}{\binom{80}{\_}}
\]

\[
\frac{\binom{20}{\_} \times \binom{60}{\_}}{\binom{80}{\_}}
\]

Probability of winning some money is ________________

Here are some other payouts from KENO for you to set-up and solve. As you write each number in the fraction, think about what that number represents in the probability.

e) 6 Spot Game
Match 6 wins $1000
Match 5 wins $50
Match 4 wins $5
Match 3 wins $1

Probability of winning some money is ________________

f) 10 Spot Game
Match 10 wins $100,000
Match 9 wins $4,000
Match 8 wins $400
Match 7 wins $50
Match 6 wins $10
Match 5 wins $2
Match 0 wins $4

Probability of winning some money is ________________