Every night at 7:00 PM on TV, the DC Lottery picks today’s winning number. The host releases three sets of ball numbered 0 to 9 into three mixing machines. After a few seconds, she captures a ball from the first machine to determine the 1st digit of the winning number. The process is repeated with the 2nd and 3rd machines to determine the 2nd and 3rd digits of the winning number. The following questions concern the DC Daily Lottery.

We surveyed a group of 100 people who bought a ticket for today’s drawing. We found:
- 16 had the 1st digit correct
- 14 had the 2nd digit correct
- 15 had the 3rd digit correct
- 5 had the 1st and 2nd digits correct
- 4 had the 1st and 3rd digits correct
- 6 had the 2nd and 3rd digits correct
- 4 had the 1st and 2nd, but missed the 3rd digit

Draw the appropriate Venn diagram for this survey.

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a) How many people had all three winning numbers? 

b) How many people had exactly none of the three winning numbers? 

c) How many people had exactly one of the three winning numbers? 

d) How many people had just two of the three winning numbers? 

e) How many people had at least one of the winning numbers?
2) Counting Principle (15 points)
   a) If you can travel from Frederick, Md. to Baltimore, Md. by car, bus or train and
   from Baltimore to London by airplane or ship, how many different ways are there to go
   from Frederick to London?

   b) An exam consists of six true or false questions. In how many ways can the exam
   be completed if leaving the answer blank is an option?

   c) A restaurant menu lists 8 appetizers, 12 entries and 5 desserts. How many different
   ways can a diner select a three course meal?

   d) Jose was told to get a dozen oranges, eight apples, and a half-pound of grapes.
   When he gets to the store he finds five varieties of oranges, five varieties of apples, and
   two varieties of grapes. Assuming that he buys only one variety of each type of fruit,
   how many different bags of fruit could he bring home?

   e) Six houses in a row are to be painted with one of the colors red, blue, green or
   yellow. In how many different ways can the houses be painted so that no two adjacent
   houses are the same color?

3) How many four-letter words (including nonsense words) can be made from the
   letters h, o, t, s, m, x, and e for each of the following conditions. (15 points)
   a) Letters can be repeated.

   b) Letters cannot be repeated.

   c) Words must begin with an h, and repetitions are allowed

   d) Words must end with a vowel, and repetitions are not allowed.

   e) Words must begin with a m and end with a t, and repetitions are not allowed.
4) a) Complete the truth table for: \( \sim p \lor \sim q \). 

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( \sim p \lor \sim q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<td>F</td>
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</tbody>
</table>

Complete the truth table for: \( \sim (p \land q) \)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( \sim (p \land q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<td>T</td>
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</tbody>
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Now compare the final results in each truth table to each other. What do you notice and how does that relate to the two compound statements? Write a few sentences explaining clearly your observation.

b) Complete the table for \( \sim [(p \land q) \oplus \sim q] \).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( p \land q )</th>
<th>( \sim q )</th>
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</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>( \sim q )</td>
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<td>T</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>( \sim q )</td>
</tr>
</tbody>
</table>
c) Let p denote “May follows April” and let q denote “June follows May.” Using the
definitions of the statements, translate each of the following symbolic expressions into
verbal sentences. (12 points)

1) \( \sim p \land \sim q \)

2) \( \sim (p \lor \sim q) \)

3) \( p \oplus q \)

Now determine the truth value of each of the statements 1), 2) and 3). Be sure to show
some work for each statements.

1) ____________ 2) ____________ 3) ____________
4) Use the data in the figure to answer each question. Show how you found each value. (15 points)

\[ \begin{align*}
\text{a)} & \quad n(A) = \underline{} \\
\text{b)} & \quad n(B) = \underline{} \\
\text{c)} & \quad n(C) = \underline{} \\
\text{d)} & \quad n(A \cup B) = \underline{} \\
\text{e)} & \quad n(A \cup B \cup C) = \underline{} \\
\text{f)} & \quad n((A \cup B \cup C)') = \underline{} \\
\text{g)} & \quad n(A \cap B \cap C) = \underline{} \\
\text{h)} & \quad n(B \cap C') = \underline{} \\
\text{i)} & \quad n(A \cap C) = \underline{} \\
\text{j)} & \quad n((A \cup B) \cap C) = \underline{} \\
\text{k)} & \quad n(B' \cap C') = \underline{} \\
\end{align*} \]