Directions: Show all necessary work in an easy to read format. It is suggested that you work each problem on your own paper and transfer your work to this sheet when you are satisfied with the answer. Answer without supporting work is not acceptable. Please staple all sheets together. Failure to follow all directions results in a 5 point penalty.

1) Let p be the statement “Arizona borders Maryland.” Let q be the statement “Arizona borders Texas.” Write the following symbolic form as an English sentence. (8 points)

a) \( p \land q \)
English statement: ________________________________

Truth value ____________________________

b) \( p \lor q \)
English statement: ________________________________

Truth value ____________________________

c) \( \sim p \land q \)
English statement: ________________________________

Truth value ____________________________

d) \( p \rightarrow q \)
English statement: ________________________________

Truth value ____________________________

2) Construct a truth table for the statement \( (p \lor q) \rightarrow (\sim q \land p) \) (10 points)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( p \lor q )</th>
<th>( \sim q )</th>
<th>( \sim q \land p )</th>
<th>( (p \lor q) \rightarrow (\sim q \land p) )</th>
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3) Let U = \{a, b, c, d, e\}, S = \{b, c, d\}, and T = \{a, c, e\}. List the elements of the following sets: (4 points)

a) \(S' \cap T = \) _________________

b) \((S \cup T)' = \) _________________

4) Let U = \{all people\}, C = \{certified public accountants\}
and E = \{self–employed people\}.
Write the following sets in English sentence form. (6 points)

a) \(C' = \) __________________________________

b) \(C \cap E = \) __________________________________

c) \(C \cup E' = \) __________________________________

5) Suppose that all of 1,000 first year students at a certain college are enrolled in a math or an English course. Suppose that 600 are taking a math course and 700 are taking English. (10 points)

a) How many students are taking both math and English?

b) Fill in the data to obtain a complete Venn diagram.
6) Of 140 students who took a math exam, 90 correctly answered the first problem, 
50 correctly answered the second problem, 
55 correctly answered the third problem, 
40 correctly answered the first and the second problem, 
30 correctly answered the first problem and the third problem, 
20 correctly answered the second problem and the third problem, 
10 correctly answered all three problems. (20 points)

a) Draw a Venn diagram.

b) How many students correctly answered the first problem only?

c) How many students correctly answered the first problem and the second problem only?

d) How many students correctly answered at least one of the problems?

e) How many students did not answer any of these problems correctly?
7) Draw a three – circle Venn diagram and shade the portion corresponding to the set. (7 points)

a) \( R \cap S \cap T' \)

b) \( R \cap (S' \cup T) \)

8) Complete the following by showing all necessary steps. (15 points)

a) A restaurant offers 5 appetizers, 8 entrees, and 5 desserts. How many ways can a person select a three-course dinner?

b) How many license plates consisting of 3 letters followed by 4 digits are possible if the letters and digits can not be repeated?
c) How many different words (including nonsense words) can be formed using the 7 letters of the word “CHARLES”?

9) A few years ago, telephone area codes in the United States and Canada consists of 3 digits. (Maryland was 301, Virginia was 703, and the District of Columbia was 202). The first digit in the area code was from 2 to 9. The second was either 0 or 1 and the third could be any digit except 0. (20 points)

a) According to these rules, how many different area codes began with the digit 3?

b) How many different area codes ended with the digit 2?

c) How many different area codes were possible?

But today with the availability of cellular phones, fax machines, pages, and modems, the conditions for the area code had to be changed so that more people would have a phone number. The first digit in the area code is from 2 to 9 and the second digit is from 0 to 8. The third digit is from 0 to 9.

d) According to these new rule, how many different area codes can begin with digit 3?

e) How many different area codes are now possible?